

Calculation- progression in maths

The following steps will outline and explain the progressive methods of calculation we introduce from EYFS to Year 2.

Understanding the concept for addition:

- Addition is increasing the total quantity/ amount.
- Addition is combining two or more groups to give a total
- Addition is the inverse of subtraction e.g. $8 + 2 = 10$, $10 - 2 = 8$
- Addition is commutative e.g. $5 + 3 = 3 + 5$. Addition can be done in any order- $12 + 16 = 28$ is the same as $16 + 12 = 28$.
- Addition is associative e.g. $5 + 3 + 7 = 5 + (3 + 7)$
- = means equal to/ the same as. Put this in different positions, e.g. $5 = 3 + 2$, $4 + 1 = 3 + 2$ (use of scales to show this to balance)

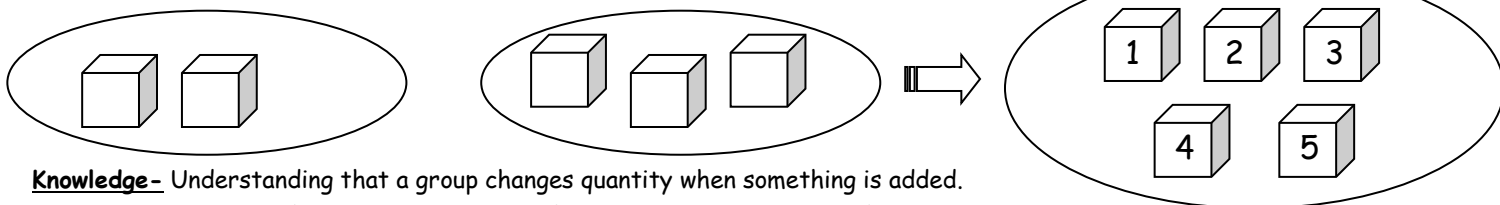
Addition using equipment

Step 1

Counting all

e.g. $2 + 3$

The child would count out 2 objects (e.g. cubes) and then 3 objects. They would find the total by putting the two sets together and counting them all. Objects including cubes are dienes provide the concrete to manipulate, count and move around.



Knowledge- Understanding that a group changes quantity when something is added.

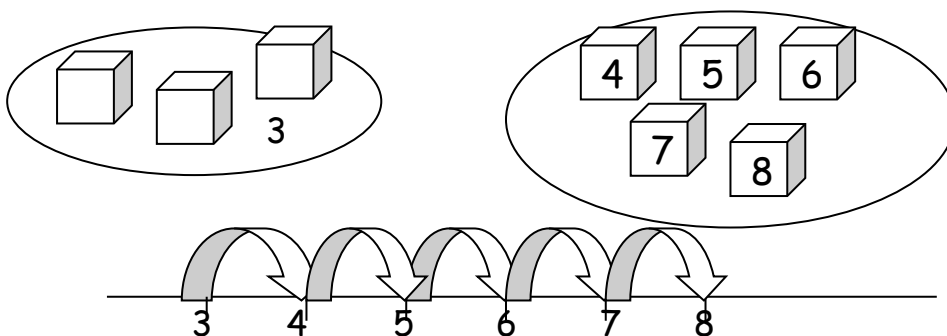
Prior knowledge- Touch counting, counting and saying number names in order.

Step 2

Counting on from the first number (Use concrete objects to count and move around).

e.g. $3 + 5$

The child would get 3 objects and then 5 more objects. The child would count on from the first number 'four, five, six, seven, eight' touching the object as they count. Extend to model this skill along a number line.



Knowledge-

- Knowing the numerical value of a quantity e.g. recognising and identifying the '4' cubes or pieces of numicon without touch counting the holes
- Being able to identify the largest quantity to begin with ideally by sight
- Understanding of symbols + and = and using words in a full sentence to explain what this means
- Beginning to see and recognise patterns within numbers and groups of numbers, e.g. 4 cubes is double 2 cubes.

Prior Knowledge- Counting aloud in order and counting on from a given number.

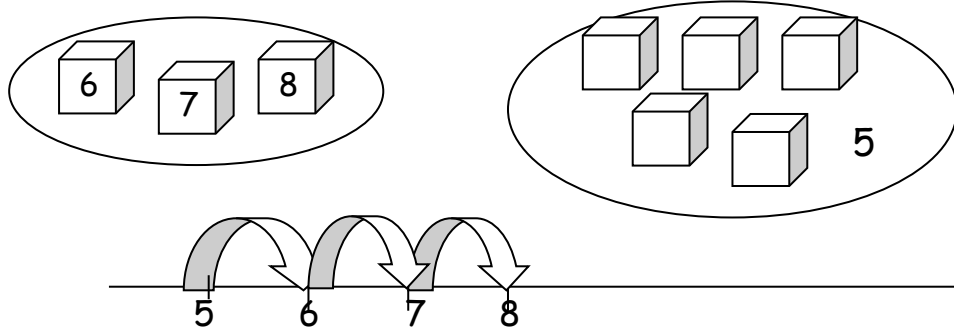
Vocabulary- count, count on, touch count, total, altogether, how many?, add, equal, same as, number names.
Misconceptions/ modelling- Children start counting the second group of objects from the beginning and start at 1 rather than continuing their counting. Model to the children putting the number in their head and count on from there. Ensure they are touching each object as they count/ say the number out loud. Encourage them to check their answer/ total by re-counting all of the objects.

Step 3

Counting on from the largest number

e.g. $3 + 5$

The child may get 3 objects and then 5 more objects. The child would choose the larger number and count on from there, 'six, seven, eight,' touching the objects as they count. This skill will be modelled on a number line.



Vocabulary- count on, touch count, total, altogether, largest, greatest, value, biggest, how many?, add, equal, same as, order, number names.

Misconceptions/ modelling- Children need to understand place value and number order to find and say which number is the greatest in value. Encourage children to recognise by quantity the largest group. Ensure they are touching each object as they count/ say the number out loud. Encourage them to check their answer/ total by re-counting all of the objects.

Knowledge-

- Knowing the numerical value of a quantity e.g. recognising and identifying the '4' cubes or pieces of numicon without touch counting the holes
- Being able to identify the largest quantity to begin with ideally by sight
- Understanding of symbols + and = and using words in a full sentence to explain what this means
- Beginning to see and recognise patterns within numbers and groups of numbers, e.g. 4 cubes is double 2 cubes.

Prior knowledge- Counting aloud in order, counting on from a given number, comparison of groups and understanding the quantity of groups.

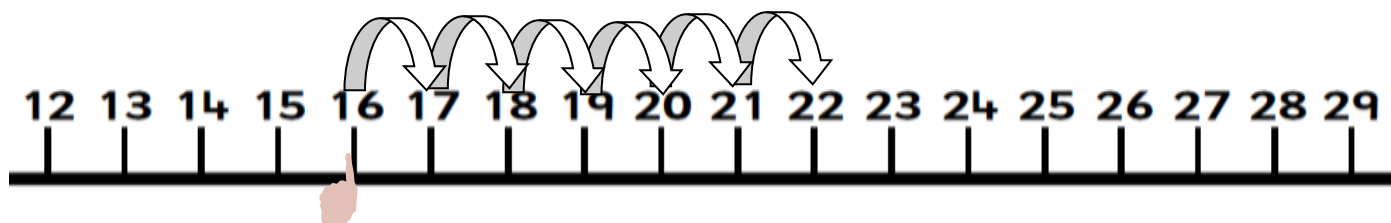
Step 4

Using a number line for addition

Children will develop their early addition skills to use a number line for solving simple addition calculations.

e.g. $16 + 6 =$

'one, two, three, four, five, six'



Vocabulary- count, count up, total, together, how many?, add, addition, equal, same as, number names, greater/ bigger in value.

Misconceptions/ modelling- Ensure the children move their finger one jump up when saying the first number to add one more. They need to understand that when adding two whole numbers the number becomes greater in value and therefore the total will be greater/ bigger.

Knowledge:

- Counting forwards in ones
- Being able to identify the largest number
- Understanding place value of tens and ones and reading 1 and 2 digit numbers
- Understanding of symbols = and + and what these mean (Children able to explain in a full sentence that adding means combining/ adding together two numbers to find how many altogether/ the total)

Prior knowledge: Understanding the value of a number and how this number represents an amount/ quantity, numeral recognition and counting in order.

Step 5

Using a hundred square for adding a 1-digit number to a 2-digit number

e.g. $48 + 7 =$ (Forty eight add on seven more, which equals/ how many altogether. Make sure the children can read this and fully understand it).

Children will put their finger on 48. They will then count across the hundred square 7 jumps remembering the jump from 50 to 51 is a jump.



Vocabulary- count, count up, total, together, how many?, add, addition, equal, same as, number names, greater/ bigger in value, one digit, two digit, tens, ones.

Misconceptions/ modelling- Children need to understand place value and have grasped a good understanding of adding one more to a number. Ensure the children move their finger one jump up when saying the first number to add one more. When moving from a tens number (e.g. 30, 40, 50), ensure children remember to make a jump to the next line to add one more (e.g. 30 to 31). When making the jumps they need to say and count the number they are adding on rather than the number they have landed on the hundred square.

They need to understand that when adding two whole numbers together the number becomes greater in value and therefore the quantity/ total will be greater/ bigger.

Knowledge-

- Counting forwards in ones
- Being able to identify the largest number and reading this number as tens and ones understanding each digit
- Locating a number on the 100 square understanding the tens and ones and where they are positioned
- Understanding of symbols = and + and able to explain this in a full sentence

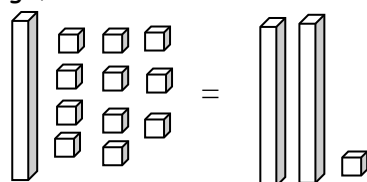
Prior knowledge- Understanding the value of a number and how this number represents an amount/ quantity, numeral recognition and accurate reading of the number, counting in order.

Step 6 Exchanging (This needs to be a step before talking about addition or subtraction).

For addition with **crossing the boundaries**, the key to transitioning between concrete methods (using blocks or counters) to formal methods such as column addition is understanding that ten ones make up one ten, ten tens make one hundred and so on. This is vital for subtraction, also. The key terms we use are 'regrouping' and 'exchanging'.

Provide opportunity for the children to visually see that a ten (tens stick/ diene stick) is the same as ten ones. To support conceptual understanding, line the ten ones up next to the ten to show they are the same size/height. Children need to then have lots of opportunity to make an exchange with the ones for a ten. This will then help the child when addition with crossing the boundary (see below).

E.g. See below. Get the children to practically move the ones and to line up ten ones next to the tens stick and to physically swap this/ 'exchange' this for a ten and count the remaining ones. Model and encourage the language the same value as, exchange, ones and tens.



Model the talk, e.g. "We know that ten ones make one ten, and we have 11 ones.... So let's exchange our ten ones for one ten. We have 2 tens which is 20 and we have one one. We have 21 altogether."

Knowledge-

- Counting forwards in ones and tens
- Understanding place value of tens and ones and can read a number with reference to the position of the digit
- Understanding that ten ones can be exchanged for one ten
- Understanding which digit represents tens and which represents ones
- Partition into tens and ones
- Count the ones first when exchanging

Prior knowledge- Counting in forwards in ones and tens (multiples of tens), reading numbers/ numerals, understanding the order of numbers and the value.

Step 7

Partitioning and recombining

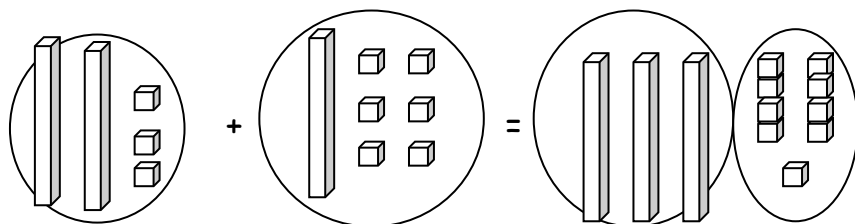
Vocabulary- partition, tens, ones, count, total, altogether, how many?, add, addition, adding on, one digit, two digit, equal, same as, number names, greater/ bigger in value, value, exchange.

Misconceptions/ modelling- Ensure children understand place value and that they can partition a number into tens and ones. Model the addition of the tens first and then the ones. Ensure they do not just add a 0 to a number when adding a ten and they understand this and how the value/ quantity gets greater in value. When using dienes understand that 10 ones is the same as a tens stick and ten tens is the same as one hundred and this can be exchanged.

e.g. $16 + 4 =$ broken down into - $10 + 6 + 4 =$ $10 + 6 = 16$ $16 + 4 = 20$

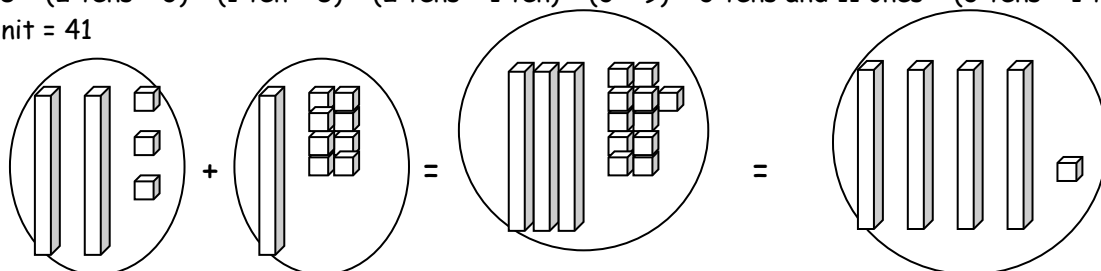
Using dienes for 2 digit addition - not crossing the boundary

e.g. $23 + 16 = (2 \text{ tens} + 3) + (1 \text{ ten} + 6) = (2 \text{ tens} + 1 \text{ ten}) + (3 + 6) = 3 \text{ tens} + 9 =$
 $30 + 9 = 39$



Using dienes for 2 digit addition - crossing the boundary

e.g. $23 + 18 = (2 \text{ tens} + 3) + (1 \text{ ten} + 8) = (2 \text{ tens} + 1 \text{ ten}) + (3 + 9) = 3 \text{ tens and } 11 \text{ ones} = (3 \text{ tens} + 1 \text{ ten}) + 1 \text{ unit} =$
 $4 \text{ tens} + 1 \text{ unit} = 41$



Ensure you talk through clearly the strategy. For example, "We are adding together twenty three, which we know is two tens and three ones, to eighteen, which we know is one ten and eight ones.

So, let's add our tens together. (Model this by moving the tens blocks side by side). Two tens add one ten is three tens.

Now let's add the ones. Three ones add eight ones is eleven ones. Can we make an exchange with our ones, for a ten?

We know that ten ones make one ten, and we have eleven ones.... So let's exchange our ten ones for one ten."

Show the pupil that you are exchanging by physically swapping your ones for a ten.

"Now we have made our exchange, is the value the same? Yes - before we have ten ones and now we have one ten. The value of both is 10. Now we have exchanged our ten ones for one ten, let's add together. Now we have four tens and 1 one, which totals forty one."

Knowledge-

- Understanding place value of tens and ones and can partition a number in tens and ones
- Understanding which digit represents tens and which represents ones
- Count the ones first when exchanging

Prior knowledge- Counting fluently in 10s, 2s and 1s, understanding that ten ones make a one ten, understanding of place value.

Step 8

Using a hundred square for 2 digit addition

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

e.g. $36 + 20 =$

Children will circle the number 36. Through their knowledge of place value, they will know that there are 2 tens in 20 and will jump down 2 tens on the hundred square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

e.g. $54 + 25 =$

Children will circle the number 54. Through their knowledge of place value, they will know that there are 2 tens in 20 and will jump down 2 tens on the hundred square. Then they will add the 5 units by jumping across the hundred square 5 jumps.

As children become fluent in this method, they will move onto a written method of addition.

Vocabulary- partition, tens, ones, count, total, altogether, how many?, add, addition, adding on, one digit, two digit, equal, same as, number names, greater/ bigger in value, counting on, multiple.

Misconceptions/ modelling- Ensure children understand place value and that they can partition a number into tens and ones. They need to be confident when counting in multiples tens and in tens from any two digit number. Ensure they do not just jump down the hundred square when adding a ten and that they fully understand this and can orally explain their reason for this. They need to understand how the value/ quantity gets greater in value. Ensure they make a jump up or down when adding the first number and that they count on the number they are adding rather than saying the number they are jumping on.

Subtraction using equipment

Vocabulary- take away, subtract, count, count back, total, how many?, left, equal, less, fewer, smaller in value.

Misconceptions/ modelling- Children need to understand that at this stage we start with the biggest number first when subtracting/ taking away and that the answer/ total will be smaller/ less in value/ quantity. Model clearly the idea of practically taking something away. They need to count how many are left when the objects have been taken away starting their counting at one to count the full group.

Understanding the concept for subtraction:

- Subtraction is the removal of an amount from a larger group (take away)
- Subtraction is the comparison of two amounts (difference)
- Subtraction is the inverse of addition
- Subtraction of one number is not commutative. For example, in addition $12 + 16 = 28$ is the same as $16 + 12 = 28$ but $16 - 12 = 4$ is not the same as $12 - 16 =$
- Subtraction is not associative e.g. $9 - 3 - 1$ is not the same as $9 - (3 - 1)$

Step 1 Taking away a group of objects

This is the method of subtraction that most of us think of first and provides a concrete opportunity for the children to practically manipulate, move and count the objects. Cubes and one digit dienes are a good resource here to use. Children when subtracting one digit numbers from a one digit number can use their fingers so that they can put a finger down when taking it away and that they can clearly visualise how many are left. Linking this to 'real life' and practical experiences is useful for the children to make links and connections.

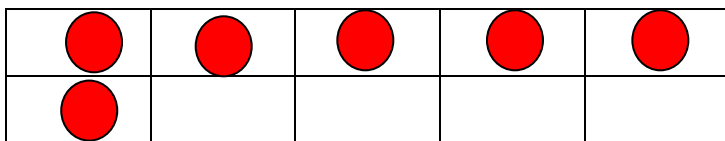
Method: The children will count out the number of objects (e.g. cubes) to match the first digit in the number sentence. They will 'take away' the second number from the set.

e.g $5 - 2 =$



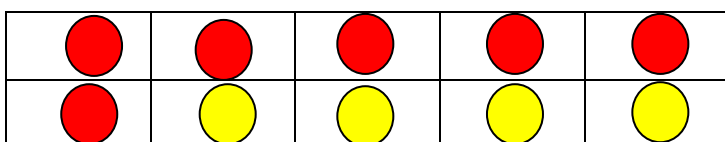
Children often use their fingers for this method to move from the concrete, to the pictorial to the abstract.

A tens frame is also useful to show 'taking away' using a concrete:



I have 10 counters. If I take 4 of them away, I will have 6 left.

At this stage, children should start making the link between addition and subtraction (related operations)



$$6 + 4 = 10 \quad 4 + 6 = 10$$

$$10 - 6 = 4 \quad 10 - 4 = 6$$

Knowledge-

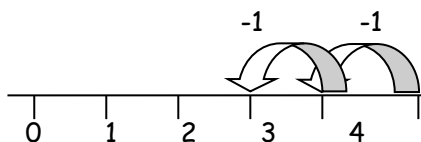
- Understanding that a group changes quantity when something is taken away
- Encourage children to visualise the value of the number (quantity)
- Knowing the numerical value of a quantity e.g. recognising and identifying that 4 counters/ cubes have the value of 4 without counting and a '4' piece of numicon without touch counting the holes

Prior knowledge- Touch counting, saying number names in order, quick fire recognition of quantity of objects/ arrays (0-5).

Step 2

Counting back on a number line

e.g. $5 - 2$



Start at 5 and count back 2 jumps

Vocabulary- subtract, subtraction, take away, count back, how many?, left, fewer, less than, number, number names, one digit, ones.

Misconceptions/ modelling- They need to understand number order and count backwards and find one less than a given number. Ensure the children move their finger one jump down when saying the first number to take one away. They need to understand that when subtracting two whole numbers in this method to put the greatest/ biggest number first and to count back/ subtract the smaller number. Children to understand that the number will become smaller in value. Adults to be careful not to cause a misconception by saying that you 'can't subtract by putting the smallest first.'

When drawing/ writing their own number lines they write/ put the starting quantity at the end of the number line.

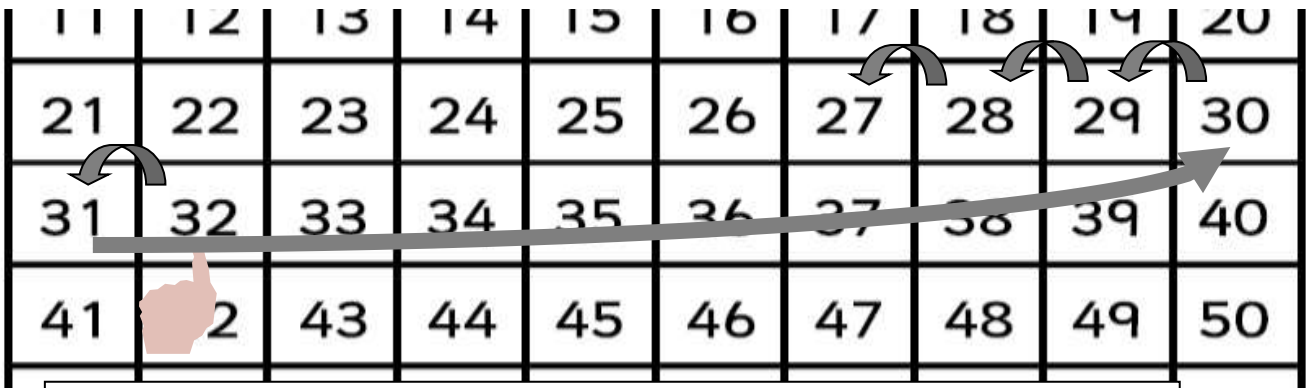
Step 3

Counting back on a hundred square

e.g. $32 - 5 =$

Vocabulary- subtract, subtraction, take away, count back, how many?, left, fewer, less than, smaller in value, number, number names, one digit, ones.

Misconceptions/ modelling- Children need to understand place value and number order and to recognise and say one digit and two digit numbers understanding the quantity/ value of the number. Model making the jump down for example from 31 to 30 and to count this as one jump down. Children to understand that the number becomes smaller in value when subtracting.



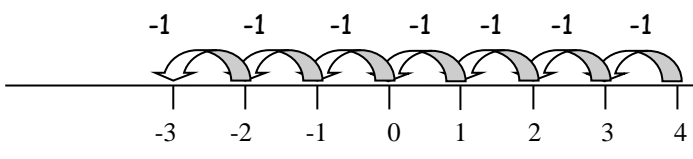
Vocabulary- subtract, subtraction, take away, count back, how many?, left, fewer, less than, smaller in value, number, number names, one digit, ones, negative, zero.

Misconceptions/ modelling- Ensure that children still understand that a negative number is smaller in value and use zero (0) as the middle point/ value. Use of a thermometer to give a real life example.

Knowledge- Being able to count backwards in ones and when counting backwards the number is smaller in value

Prior knowledge- Touch counting, saying numbers in order, recognising numbers.

Counting back into negative numbers on a number line



Start at 4 and count back 7 jumps.

Step 4

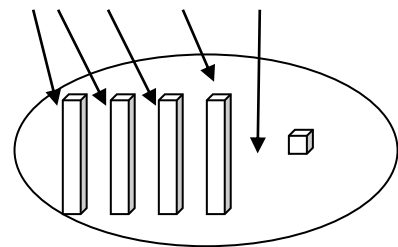
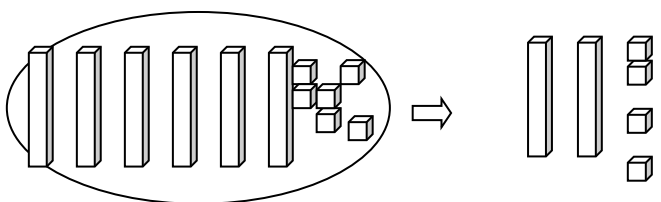
Taking away a 1 or 2-digit number from another 2-digit number

The take away method is adapted when using two digit numbers:

e.g. $65 - 24 =$

Vocabulary- subtract, subtraction, take away, count back, how many?, left, fewer, less than, smaller in value, number, number names, one digit, two digit, tens, ones, partition, split, place value, decrease

Misconceptions/ modelling- Ensure that children understand place value and that a two digit number is made up of tens and ones. Ensure they understand that when subtracting that the number becomes smaller in value and that they can see this by physically taking the number/ quantity away using the dienes.



Remove 24 and re-count what is left

Knowledge-

- Counting backwards in ones
- Partition numbers into tens and ones
- Know and quickly derive number bonds up to and including 10 and subtraction facts

Prior knowledge-

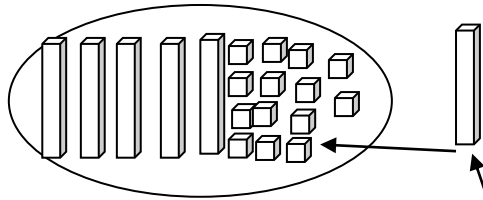
- Understanding of one and two digit numbers and partitioning, understand the quantity of numerals, ability to visualise a quantity.

Step 4 b. (Step 6 in addition is crucial prior to this step- see above).

They need to understand how to exchange 10 ones for a 10 stick.

E.g. $64 - 25 =$

The children need to visualise how physically this can be taken away. Encourage them to exchange a ten stick for ten ones as they understand this is the same value.



Exchange the ten for ten ones and then the children can physically take away the 2 tens and 5 ones (25) from the group and count how many are left.

Step 5

Subtracting 2-digit numbers using a hundred square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

e.g. $72 - 30 =$

Children will circle the number 72. Through their knowledge of place value, they will know that there are 3 tens in 30 and will jump up 3 tens on the hundred square when subtracting.

Vocabulary- subtract, subtraction, take away, count back, how many?, left, fewer, less than, smaller in value, number, number names, one digit, two digit, tens, ones/ units.

Misconceptions/ modelling- Ensure that children understand place value and that a two digit number is made up of tens and ones/ units. They need to be able to confidently partition numbers. Ensure they understand that when subtracting that the number becomes smaller in value and that they do not just learn to take the first digit away from the other.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

e.g. $87 - 23 =$

Children will circle the number 87. Through their knowledge of place value, they will know that there are 2 tens in 20 and will jump up 2 tens on the hundred square. Then they will subtract the 3 units by jumping back on the hundred square 3 jumps.

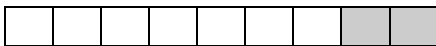
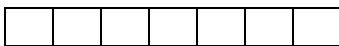
As children become fluent in this method, they will move onto a written method

Step 6

Subtraction as 'Finding the Difference'

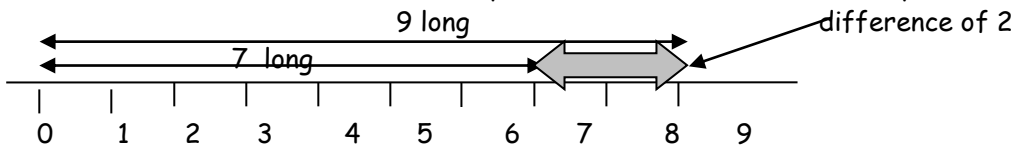
Method: Step 1: The children would begin by comparing towers of cubes of different heights/lengths and counting the difference between them. Using this bar model shows clearly that the word 'more' in the question refers to the gap or difference between the objects/ quantities. Double sided coloured counters are a good way of showing to find the difference.

e.g. $9 - 7 =$



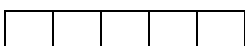
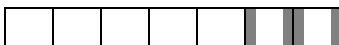
9 has 2 more cubes.
I would need 2 more cubes to make them the same length.
9 and 7 have a difference of 2

This would be shown on a number line by the difference in the number's position.



Wordy problems: Max has 7 sweets and Sam has 5 sweets.

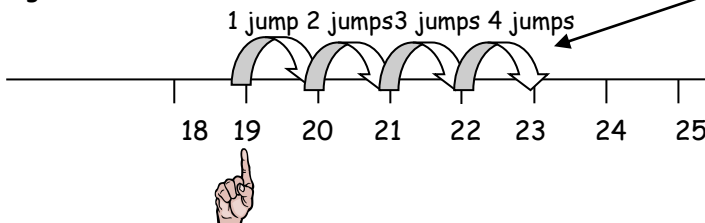
How many more sweets does Sam have compared to Max?



Difference of 2. Max has 2 more.

Step 2: The children will put their finger on the number being subtracted and count how many jumps it takes to reach the original number.

e.g. $23 - 19 =$



difference of 4

Vocabulary- subtract, subtraction, take away, count back, how many?, left, fewer, less than, smaller in value, number, number names, one digit, two digit, tens, ones, find the difference.
Misconceptions/ modelling- Children need to understand that addition and subtraction are the inverse. 'Find the difference between' means to carry out a subtraction. Often questions will be worded like 'How many more/fewer is... Ensure they understand the word more in this context and that this doesn't mean adding altogether.

Written methods for addition and subtraction

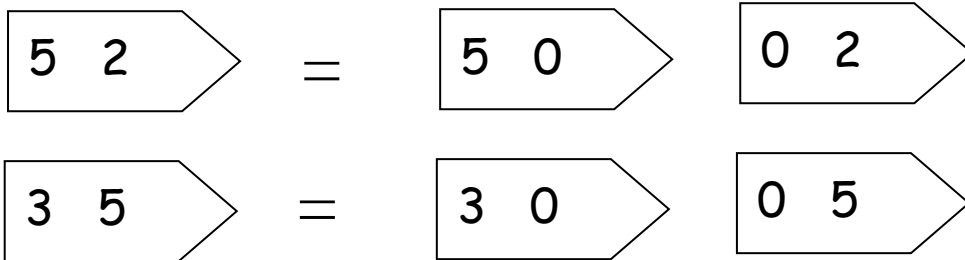
Once the child becomes confident in addition and subtraction using equipment, they can move on to written methods.

Addition- Step 1

Addition using partitioning (without crossing the tens boundary)

e.g. $52 + 35$

Step 1: Split the numbers into tens and ones



Step 2: Add the tens together first

$$50 + 30 = 80$$

Step 3: Add the units together

$$2 + 5 = 7$$

Step 4: Add you two totals together to find your final answer

$$80 + 7 = 87$$

In their books, their written calculation should look like this:

$$52 + 35 =$$

$$50 + 30 = 80$$

$$2 + 5 = 7$$

$$80 + 7 = 87$$

Step 2

Addition using partitioning (crossing the tens boundary)

e.g. $46 + 27 =$

Step 1: Split the numbers into tens and ones

$$46 + 27 =$$

$$40 + 20 =$$

$$6 + 7 =$$

Step 2: Add the tens together first

$$40 + 20 = 60$$

Step 3: Add the units together

$$7 + 6 = 13$$

This will then need to be partitioned into tens and units to aid the calculation of the totals

$$13 = 10 + 3$$

Step 4: Add your two totals together to find your final answer

$$60 + 10 + 3 = 73$$

In their books, their written calculation should look like this:

$$46 + 27 =$$

$$40 + 20 = 60$$

$$6 + 7 = 13$$

$$60 + 10 + 3 = 73$$

Step 3

Addition using the column method (without crossing the tens boundary)

Children will be shown to set out their number sentence like this:

$$\begin{array}{r} 34 \\ +42 \\ \hline \end{array}$$

They will be asked to calculate their answers in the same way they did when using the partitioning method:

$$\begin{array}{r} 34 \\ +42 \\ \hline 70 \text{ (30+40)} \\ \underline{6} \text{ (4 + 2)} \end{array}$$

Finally, the children will add their final answers together:

$$\begin{array}{r} 34 \\ +42 \\ \hline 70 \\ \underline{6} \\ \hline 76 \end{array}$$

Step 4

Addition using the column method (crossing the tens boundary)

They will set it out in a similar way :

$$\begin{array}{r} 67 \\ +26 \end{array}$$

Again, they will add the tens together first and then the units:

$$67$$

+26
80
13

They will now add the two answers together to find the final answer:

67
+26
80
13
93

The children are then encouraged to try the addition by adding the ones first and then the tens. Do we get the same answer?

48	48
<u>+34</u>	<u>+ 34</u>
70 (40 + 30)	12 (8 + 4)
<u>12 (8 + 4)</u>	<u>70 (40 + 30)</u>
82	82

This can then be developed by adding 3 digit numbers.

Step 5

Column method involving crossing the ten

e.g. $37 + 28 =$

Step 1: Set out your addition number sentence in the column method:

37
+28

Step 2: First, add the 7 and the 8 to give 15. Because we understand our number system we know that this means one ten and five ones. We need to put the 5 under the ones but put the 1 ten under the tens column at the end like this:

37
+ 28
5
1

Step 3: Next. Add the tens remembering to add the one ten that is underneath the tens column:

37
+28
65
1

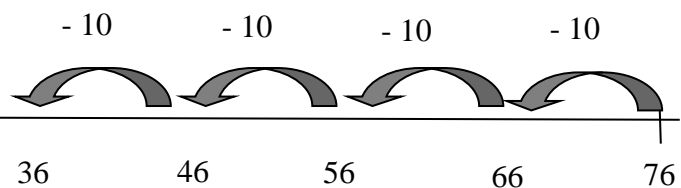
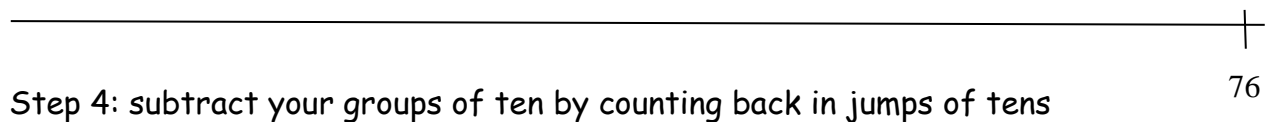
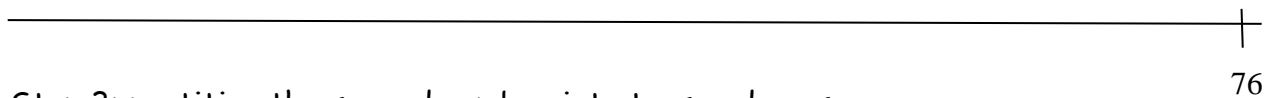
Subtraction- Step 1

Subtracting groups of ten using your own number line

e.g. $76 - 40 =$

Step 1: Draw your own number line

Step 2: Put the largest number at the end of your number line



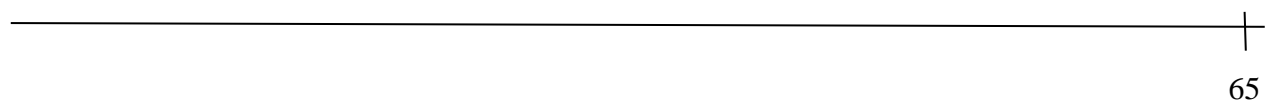
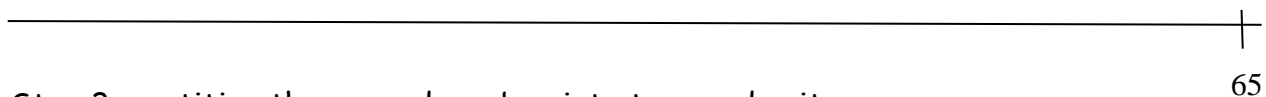
Step 2

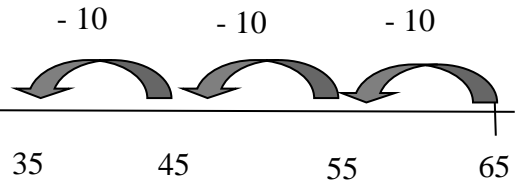
Subtracting groups of tens and units by drawing your own number line

e.g. $65 - 34$

Step 1: Draw your own number line

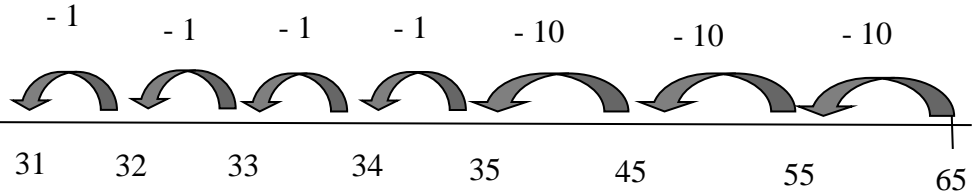
Step 2: Put the largest number at the end of your number line





Step 5: subtract your units by counting back in

ones



Step 3

Subtraction using the column method (without crossing the tens boundary)

They will set it out in a similar way to addition column method :

$$\begin{array}{r} 67 \\ -26 \end{array}$$

Again, they will subtract the tens together first and then the ones:

$$\begin{array}{r} 67 \\ -26 \\ 40 \\ 1 \end{array}$$

They will now add the two answers together to find the final answer:

$$\begin{array}{r} 67 \\ -26 \\ 40 \\ 1 \\ 41 \end{array}$$

Vocabulary- Odd, even, multiply, multiplication, times, repeated addition, groups of, array, equal, lots of, lots, larger, quantity, rows of, product.

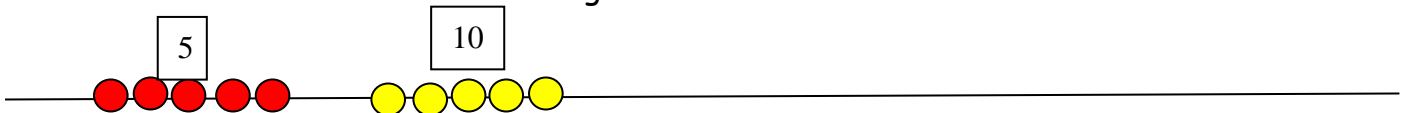
Multiplication

Understanding the concept for multiplication:

- To multiply is to add groups of numbers. Multiplication is repeated addition, e.g. $5 \times 2 = 10$ is the same as $5 + 5$, $2 + 2 + 2 + 2 + 2$
- Multiplication means the end number/ end product will be greater/ larger.
- Multiplication can be done in any order (commutative). $2 \times 3 = 6$ this is the same as $3 \times 2 = 6$
- Multiplying by 2 is the same as doubling ($4 \times 2 = 8$ is the same as Double 4, $4 + 4 = 8$)
- The relationship between multiplication and division. For example, $5 \times 2 = 10$, $10 \div 2 = 5$

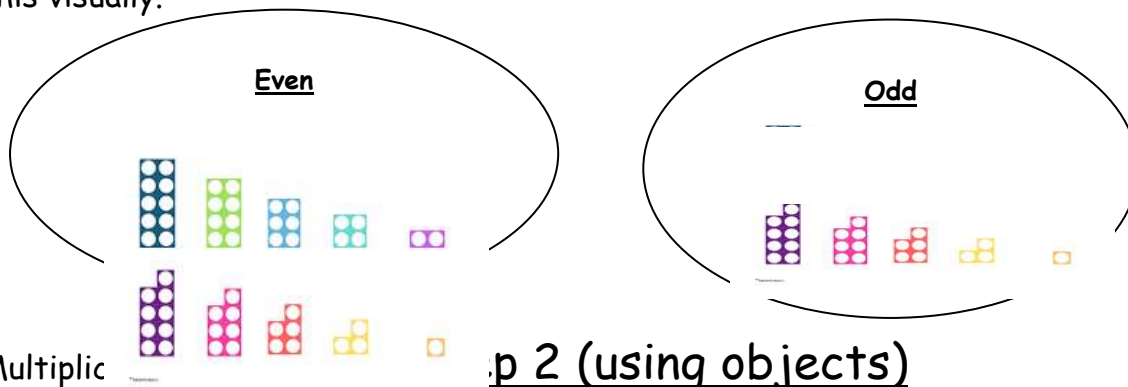
Step 1. The basics required- Counting in steps of 2, 3, 5 and 10

Children need to be confident in counting in steps forwards and backwards from any given number. They need regular practise using concrete and visual objects to visualise this. Hundred squares are a good visual resource to count forwards and backwards in tens. Strings with beads on, double sided coloured counters, counting sticks and pegs on a washing line/ clothes hanger are all good visual resources. Children need to be able to recall from memory multiplication facts for the 2, 5 and 10 times tables. They need to recite these in order. They need to make the link between doubling and $\times 2$ tables.



Link counting in multiples to real life examples such as money counting in 2ps, 5ps, 10ps and 20ps and on a clock face counting in 5 minutes. They need to recognise the patterns in the multiplication, e.g. all numbers in the $2 \times$ tables are even, all numbers in the 5s end with 0 and 5, in the $10 \times$ table end with 0.

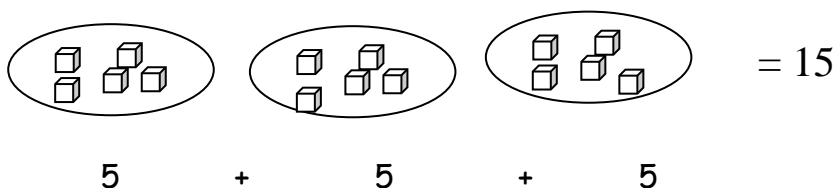
They need to understand the structure of numbers between odd and even numbers and see this visually.



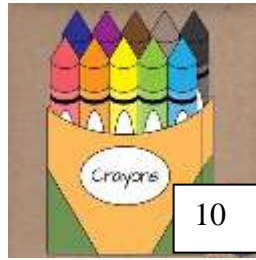
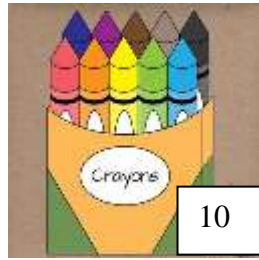
Multiplic

p 2 (using objects)

Children need to use groups, arrays and number lines. They need to understand that multiplication is **repeated addition**. Children will be encouraged to use practical objects such as cubes or counters to make 3 groups of 5 and then count the total.



Use of real life objects at this stage is crucial to visualise 'groups of'.

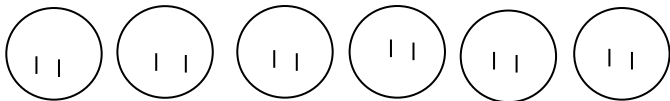


3 groups of 10. $3 \times 10 = 30$

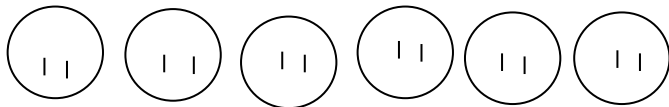
Step 2 (using a written method)

Multiplication as repeated addition. Children need to have a secure understanding of repeated addition for multiplication and lots of opportunity to practise this before moving on. They need to develop an understanding that multiplication and division are inverse operations.

e.g. $6 \times 2 =$ There are 6 groups of 2. 6 multiplied by 2 is 12. Multiplication can be done in any order. Children will know that this means 6 groups of 2. They will draw 6 circles and put 2 lines in each circle.

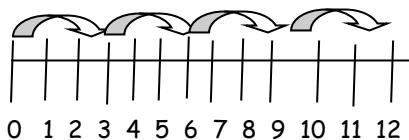


Now write the value of each circle under the circle



$2 + 2 + 2 + 2 + 2 + 2 = 12$ Now find the total.

+3	+3	+3	+3
----	----	----	----



Counting in steps using jumps on a number line

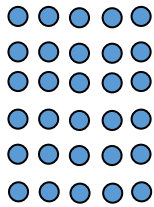
This is then moved on by encouraging the children to use their knowledge of multiples to write the numbers in the groups rather than the pictorial. For example. $3 \times 5 = 15$



Step 3 Arrays

Multiplication as arrays

Children can also use arrays to help solve multiplication problems. The term 'array' means arranging objects or dots into a grid. For example, 6×5 will mean 6 rows of 5. Children will show their working like this:



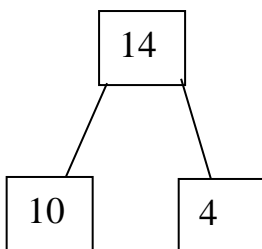
Use of real life objects to show arrays



$2 \times 3 = 6$ (2 rows of 3 eggs), $3 \times 2 = 6$ (3 rows of 2 eggs)

Making links and using this to partition- Children need to know their doubles and make the link between multiplying by 2 to doubling.

For example, Double 14 (This is the same as 14×2 , 14 2 times) = Double 10 + Double 4



Double 10 is 20. Double 4 is 8. $20 + 8 = 28$
Dienes is a good resource to show this using objects/ visuals.

Division-

Understanding the concept for division:

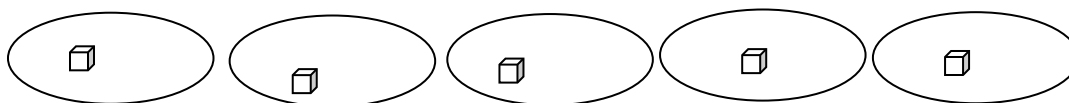
- Division is sharing into equal parts or groups. It is the result of "fair sharing".
- Division is not associative and cannot be done in any order (the order in which this is done changes the outcome/ product).
- Division- (At this stage) biggest number goes first to get a whole number.
- Division is the same as repeated subtraction, e.g. $6 \div 2 = 3$ can be read as how many groups of 2 can be taken away from 6
- Division is the inverse of multiplication, for example If $3 \times 4 = 12$, what is $12 \div 4 =$ (Answer is 3)
- Division by 2 is the same as half of ($\div 2$), division by 4 is the same as a quarter of

Vocabulary- Equal groups of, divide, divided by, divided into, shared into, half, quarter, share, group

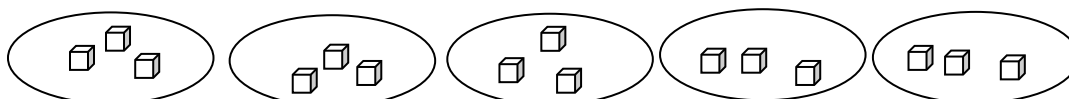
Step 1 (using objects to share) Sharing

Sharing objects and counters out **equally**. They need to understand the term equal and what it looks like with equal parts. For example, they need to visualise equal parts of an object, shape or quantity, for example equal parts of a cake/ pizza. The early link between fractions can also be made here, for example sharing into four equal parts and finding one part would be the same as finding one quarter.

For example, Sharing 15 objects into 5 groups



I have put one cube in each group. Now I will put a 2nd in.



There are 3 cubes in each group so 15 shared equally into 5 is 3.

Step 2 (using a written method)- Grouping

When grouping, children need to count the number of groups they have. For example, $15 \div 3 = 5$ can be also worded as "How many groups of 3 are there in 15?"

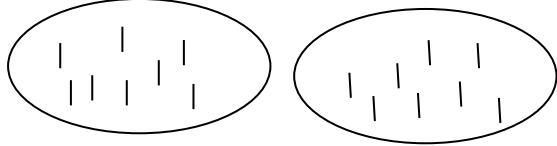
Using beads on a string or counters and practically moving these to group them together would be a good visual for the children to be provided with the concrete.

This would then be moved onto division for groupings using drawings (pictorial)

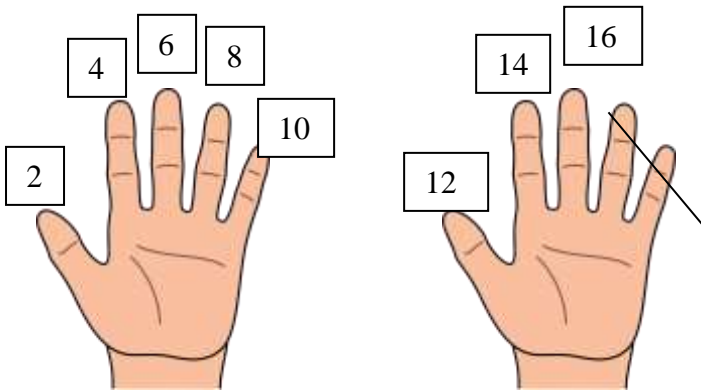
e.g. $16 \div 2 =$

This means I am sharing 16 into 2 groups

There are 8 in each group so $16 \div 2 = 8$. 16 divided into 2 groups is 8. There are 8 in each group.



Children can then move on to less concrete methods using fingers to count how many groups of 2 are there is 16. Count up in 2s until get to 16 and then count how many fingers/ times counted which is 8.



Step 3- Division as arrays and bar models



$6 \div 3 = 3$
(6 objects divided/ grouped into 3 rows)

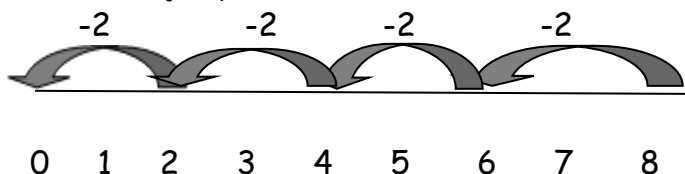
25

Bar models- Amy has 25 marbles and has 5 jars. He puts the same in each jar. How many go in each jar?



Step 4- Understanding as repeated subtraction

Children need to make the connection with division as repeated subtraction. For example, $8 \div 2$ can be read as how many groups of 2 can be taken away from 8? 4 groups of 2 (count the number of jumps)



Using and applying these calculation skills

RUCSAC

'RUCSAC' is a mnemonic which outlines a methodical approach for solving word problems under the four operations. 'RUCSAC' stands for:

R - read the questions

U - underline the important parts e.g. numbers or vocabulary that might give us a clue

C - choose your calculation

S - solve your answer (using the desired calculation strategy)

A - write your answer

C - check your answer by doing it all again in a coloured pencil

